Mathematical Approach Methodology to Analysis and Design of LPS

Rolling Sphere Method

Biagione Rangel de Araújo
BRC – Biagione Rangel Consultoria
Natal, Brazil
contato.brc@biagione.com.br

D.Sc José Tavares de Oliveira
Department of Electrical Engineering
UFRN – Universidade Federal do Rio Grande do Norte
Natal, Brazil
jtavares@ct.ufrn.br

***Abstract* - This paper presents the results of some analysis in design of LPS - Protection System Lightning using a mathematical approach methodology from the areas of knowledge of plane geometry and trigonometry, where we used the rolling sphere method.**

Keywords— mathematical approach; analysis methodology; design LPS; analysis LPS; limits LPS

I. Introduction

This methodology was developed to support the analysis and emission of technical reports of LPS installed in storage stations and transfer of flammable fluids.

There is a history of accidents of lightning striking storage tanks of these fluids. These accidents occurred as much in stations equipped with or without LPS.

In the process of analyzing projects and the conformity assessment of these on site of installations, it was identified that the vertical and horizontal graphical representations did not provide elements which could provide a conclusive opinion on the effectiveness of the project.

The vertical graphics (vertical cross-sectional), usually with two sectional drawing, didn't provide an adequate visualization of protected volume that installed LPS provides, since the cross-sectional drawing does not always correspond to the most critical situation, because it's not a representation of a 360 degree rotation of the rolling sphere.

The graphical representations of the horizontal projections correspond to the coverage at ground surface (limiting the protected area), therefore not including the height projection dimensions of the structures under the protection of the LPS.

In the calculation report phase, the need for a tool to calculate the height dimensions of the LPS became evident, as a function of given height and horizontal distance required to protect a structure, replacing the graphical method of trial and error.

In an attempt to obtain a tool to meet the identified need, a comprehensive search of mathematical support was performed, in the standards for the Installation of LPS: Brazilian NBR-5419 [1], International IEC 62305-3 [2] and American NFPA-780 [3]. In the analysis of the reference standards, we identified two basic mathematic equations. In the IEC 62305-3 [2] for the case of two wire air-termination system or air-termination: equation (1), to calculate the penetration distance of the rolling sphere. In the NFPA-780 [3] the equation (2), to calculate the horizontal protected distance:

$$p = r - [r^2 - (d / 2)^2]^{1/2} \qquad (1)$$

Where:
p penetration distance of the rolling sphere;
r radius of the rolling sphere,
d distance separating two parallel air-terminal horizontal wires or two air-terminal rods.

$$d = [h_1(2R - h_1)]^{1/2} - [h_2(2R - h_2)]^{1/2} \qquad (2)$$

Where:
d horizontal protected distance
h1 height of the higher mast
R rolling sphere radius
h2 height of the lower mast:

It can be seen that where h2 is equal to zero, we get the horizontal distance of the cover at the level of the reference ground (ground surface). With this consideration, we obtain equation (3).

$$d = [h(2R - h)]^{1/2} \qquad (3)$$

Where:
d horizontal protected distance
h height of the wire air-termination system or air-termination
R rolling sphere radius

II. Definitions

a) Air-termination rod: Metallic elements such as rods intended to intercept lightning flashes*;*

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Company e-mail: contato.brc@biagione.com.br
Home: www.biagione.com.br .

b) *Coverage radius (rc):* The distance between the point of the cover margin, of a determined envelopment of a LPS. This distance determines the size of horizontal projection of the fictitious plane, given by equation (2);

c) *Cover margin height (hc)*: It is the dimensions of the height of the nearest point on the envelope over the structure under protection of the LPS*;*

d) *Distance from the critical point (a):* The distance between the critical point and one metallic element such as rods or catenary wires*;*

e) *Envelopment:* Geometric shape that limits the protected volume according to rolling sphere method;

f) *Coverage margin (cm):* It's the shortest distance between a point of the structure under protection of the LPS and the envelopment of the protective volume. The dimension of the margin corresponds to the perpendicular tangent measurement of the envelopment to the nearest point of the structure under protection. The graphical representations of "***cm"*** are: Fig.1.a; Fig1.b and Fig.1c, contained in Fig.1.

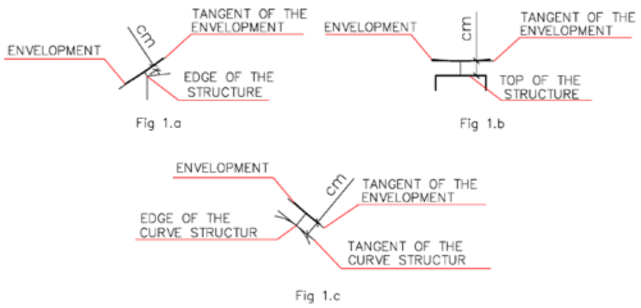

Fig. 1. Examples of coverage margins

g) *Critical point (Cp):* The lowest point of the radius of the rolling sphere, supported by two LPS, from the ground or reference level*;*

h) *Fictitious plane (Fp):* Assumed horizontal plane that provides coverage of protection at a given height*;*

i) *Hazardous areas:* Area surrounding storage facilities or transfer station flammable liquids or gases, due to the possibility of containing flammable or explosive mixtures. They are defined in: Zone 0 (when the explosive mixture / inflammable is still there or for long periods); Zone 1 (likely to occur in normal operating conditions) and Zone 2 (it's unlikely - abnormal condition of operation);

j) *Height of the critical point (hcp)*: This is the dimension of the height of critical point*;*

k) *LPS:* Lightning Protection System;

l) *Protection zone (Zp):* The volume delimited by the rotation or extension of the geometric figure bounded by the rolling sphere radius around an air-termination rod (isolated rod) or along the same, in the case of a wire air-termination.

m) *Surface coverage:* The horizontal projection of the LPS at ground level (ground surface) The distance is given by equation (3)*;*

n) *Wire air-termination:* Catenary wires intended to intercept lightning flashes

III. Boundary Conditions

A. For calculation and design

The boundary conditions presented here are those determined for the examples of this paper:

- The coverage margin determined for facilities and buildings is 1.0m and 2.0m for hazardous areas;
- All parts of the external LPS (air-termination and down-conductors) shall be at least 1 m away;
- The protection levels are those defined in IEC [2], for rolling sphere radius R: 20 m for Level I; 30 m for Level II; 45 m for Level II and 60 m for Level IV;
- Protected horizontal maximum distances are given by equation (3), and correspond to: 20.0m for Level I; 30.0m for Level II; 45.0m for Level III and 45.0m for Level IV;
- The distance from the critical point to any of the axes of an air-termination rod or wire air-termination rod must be less than the level of protection of the rolling sphere radius defined in the design.

B. Boundary limit Conditions

The boundary limit conditions define the smallest coverage margin for the design, in the case of dimensional variations:

- The coverage margin for hazardous areas is 1.0 m (the IEC [2] in item D.5.1);
- The IEC [2] as determined in item E.5.2.2.2, that the penetration distance '**p**' should be less than '**ht**'(height of the LPS) minus the height of objects to be protected. As a safety margin, we adopted the value of 0.1m. The equation (1) determines the penetration sphere radius;
- The protection levels are the same as those used for the design, mentioned in the boundary conditions of the item A;
- The distance from the critical point to any of the axes of an air-termination rod or wire air-termination rod must be less than the level of protection of the rolling sphere radius defined in the design.

IV. System of Units

The units used are the International System-SI (MKS), however, the dimensions shown in the figures are in millimeters and the data in the tables are in meters.

V. ANALYSIS OF IMPLEMENTED PROJECTS

A. Building protection by wire air-termination rod

The analysis consists of the appraisal of the real effectiveness of LPS in the configuration shown in Fig.2, using the equation (2). The summary of the calculations are contained in Table I and the graphical representation of the fictional protection plane is in Fig. 3.

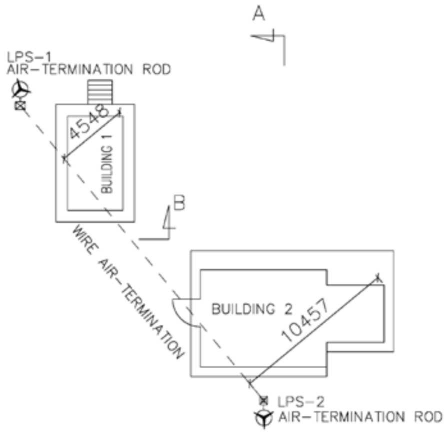

Fig. 2. Example of wire air-termination for protection of 2 buildings

TABLE I. RESULTS OF LPS RADIUS CALCULATION FIG.2

Designation	**Data**	
Installation under protection	Building 1	Building 2
Design protection level	I	I
Height of LPS Fig. 2 (m)	12.440	
Height of the building of Fig. 2 (m)	3.50	4.50
Limit coverage margin (m)	0.10	0.10
Lateral distance from the LPS axis (m)	4.60	10.50
Radius of the fictitious plane (m)	7.069	5.755
Radius of the fictitious plane in air-termination rod (m)	7.212	5.898

As noted, the LPS doesn't provide an effective of Level I coverage for the building 2, as set out in the project, since the calculated distance (5.755m) is less than the required coverage (10.50m). The graphical representation of the calculated coverage plan is in Fig. 3.

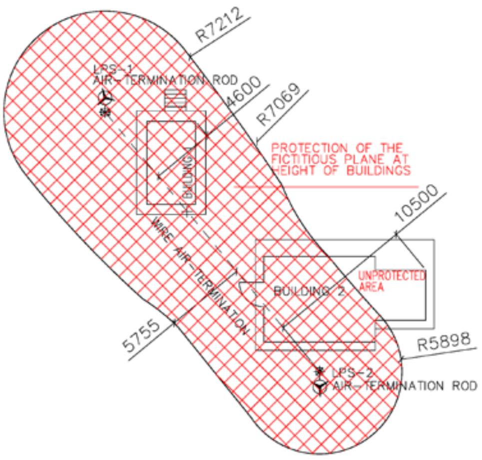

Fig. 3. Projection of horizontal coverage for the LPS of Fig. 2

Considering the mathematical approach methodology cited, it is evident that this design only provides cover against lightning for Level III, because the coverage radius calculated is 11.24m (bigger than the required 10.50m). The radius calculated for Level II is 8.36m.

B. *Analysis of LPS design for a building*

In this section we develop an analysis of an existing design, using the equations (2) and (3) and the calculation of margin coverage, with the mathematical approach methodology, the object of this paper. The input data are: the dimensions of the building, the height of LPS and the distance from the edge of the building from the axis of LPS. The results of the calculations are shown in TABLE II. Column 1 shows the results from the calculation of the margin of coverage radius by the equation (2) and the m)Surface coverage by equation (3), for the design data. Column 2 shows the limits, considering a minimum coverage margin of 0.1 m. This boundary condition indicates that the project can support possible changes in the implantation of a reduction in the height of the LPS of up to 0.2m, and in the side distance in relation the design position up to 0.44m. The graphic representations are in Fig. 4 and Fig. 5.

TABLE II. RESULTS OF THE DESIGN AND LIMITS CALCULATIONS

Designation	**Data**	
Installation under protection	Column 1	Column 2
Design protection level	I	I
Height of LPS - Air-termination rod (m)	16.00	15.80
Structure height to protect (m)	6.00	6.20
Side distance from the cable axis (m)	4.50	4.94
Radius of the fictitious plane by equation (2) (m)	5.313	5.078
Delta horizontal distance by equation (2) (m)	0.813	0.138
Coverage margin - Fig 1a (m)	0.589	0.100
Horizontal protected distance (ground surface) (m)	19.596	19.554

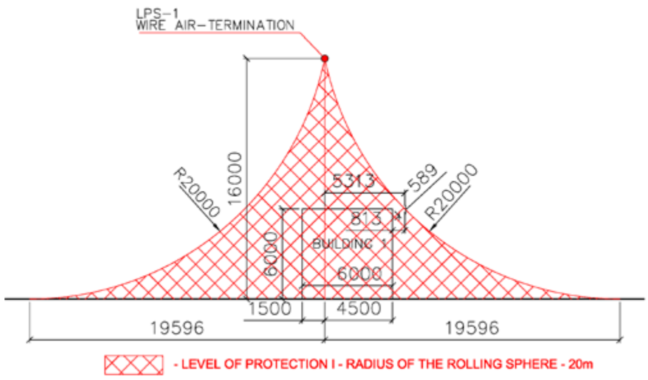

Fig. 4. LPS by wire air-termination rod- Column 1- design representation

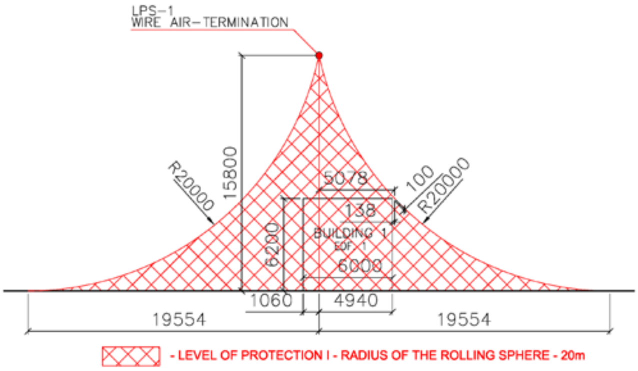

Fig. 5. LPS by wire air-termination rod - Column 2 - limits representation

The coverage margin calculated by the mathematical approach methodology and ratified in the graphic representation (Fig. 4) is 0.589m, but if it is calculated with equation (2), the value is 0.81m; therefore, there is a difference of 0.224m. In assessing the limits, these values are 0.10 m and 0.138 m, respectively (Fig. 5).

C. Protection of an installation with hazardous area

This section reports the results of an analysis by the mathematical approach methodology of one installation with a hazardous area. The calculation results are shown in TABLE III. Col.1 (Column 1) and Col.2 (Column 2) correspond to values considering isolated mast (air-termination rod), being that Col. 1 corresponds to the design condition and Col.2 is an assessment of the effectiveness by increasing the LPS quota. Col. 3 (Column 3) indicates the design option (replacing the LPS of the air-termination rod with a wire air-termination rod) to make protection of the LPS effective for installation. Col. 4 (Column 4) is the results of the calculation of the limits of the option of Col. 3. Fig. 6 is the graphical representation of Col.1 and Fig.7 to Col.3..

TABLE III. RESULTS OF LPS CALCULATIONS FOR HAZARDOUS AREA

Designation	**Col. 1**	**Col. 2**	**Col. 3**	**Col. 4**
Height of LPS (m)	18.00	20.00	20.00	18.00
Design protection level	I	I	I	I
Side distance of the hazardous area center in relation to the LPS	7.00	7.00	3.085	4.611
Center height of the hazardous area	1.50	1.50	1.50	1.50
Radius of the hazardous area	3.00	3.00	3.00	3.00
Horizontal projection (ground level)	19.900	20.000	19.900	19.900
Coverage margin (Fig.1c)	- 0.447	- 0.389	2.000	1.000
Radius of the upper intersection point of the envelopment with the hazardous area (input)	7.276	7.392		
Radius quota at the higher point of intersection	4.487	4.474		
Radius of the lower intersection point of the envelopment with the hazardous area (output)	9.708	9.665		
Radius quota at the lower point of intersection	2.792	2.877		

As can be seen, the negative values for Col. 1 and Col. 2 coverage margins indicate that the envelopment enters the hazardous area, so the isolated air-termination rod doesn't provide adequate coverage of the Level I for this installation. However, considering the configuration of Col. 3 (wire air-termination rod) this coverage is effective. So the solution is to replace LPS for a configuration using wire air-termination rod. Col. 4 indicates the limit of installation for the LPS, wire air-termination rod from the center of the external hazardous area (reference for measurement of margin coverage), provided that the other dimensions are ensured.

Fig. 6 is the graphical representation of Col. 1 and Fig. 3 is Col. 3. The graphical representations for Col.2 are not shown, due to this option not providing adequate coverage to the installation and Col. 4 is similar to Fig. 7.

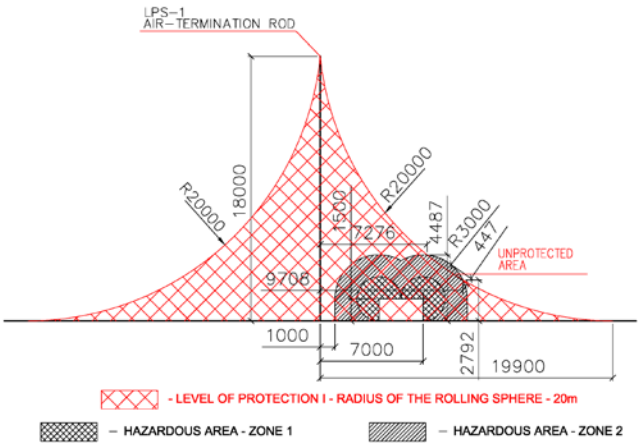

Fig. 6. Graphical representation of Col. 1, for TABLE III

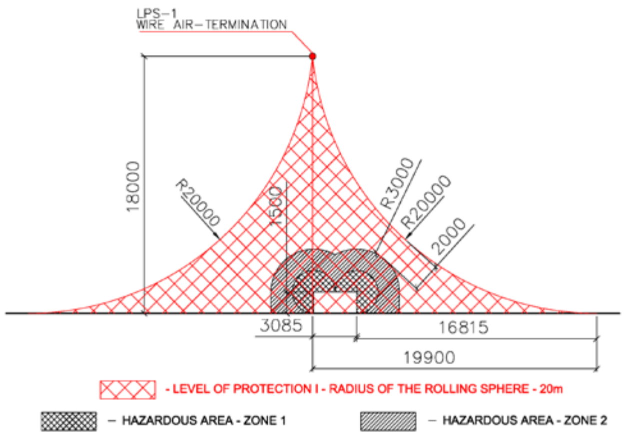

Fig. 7. Graphical representation of the Col. 3, of the TABLE III

VI. LPS DIMENSIONING

A. Wire air-termination rod at different height

This example has the premise that there is a wire air-termination rod at a height of 19.6m. So it shows the calculation to another parallel wire air-termination rod, in order to integrate the existing LPS for protection of an installation, with a height of 9.5m for a distance between the LPS of 30.0m. The input data are: height of the installation; coverage margin;

height of the existing sensor cable and distance between the LPS. The results of calculations from the mathematical approach methodology are presented in Table IV and the graphical representation for the Protection Level I in Fig. 8.

TABLE IV. RESULTS OF THE CALCULATIONS OF 2 LPS AT DIFFERENT HEIGHTS

Designation	**Data and calculated values**			
Design protection level	I	II	III	IV
Installation height to protect	9.50	9.50	9.50	9.50
Project coverage margin	1.00	1.00	1.00	1.00
Distance between the LPS	30.00	30.00	30.00	30.00
Height of the existing LPS	19.60	19.60	19.60	19.60
Height of the lowest LPS calculated	15.502	11.723	10.591	10.526
Height of the critical point	10.500	10.500	10.500	10.500
Distance from the critical point to the lowest LPS	13.231	8.478	2.867	-1.768
Height in the 5m radius, from the smallest LPS	12.272	10.702	10.551	10.883
Height in the 5m radius, from the highest LPS	14.329	15.459	16.319	16.802
Horizontal protected distance of the highest LPS (ground surface)	19.996	28.140	37.146	44.360
Horizontal protected distance of the lowest LPS (ground surface)	19.488	23.790	29.001	33.946

As can be noted, the design for the Protection Level III and IV require an equivalent quota for the LPS, and also for Level IV, the critical point is outside of the boundaries between the LPS (negative value). For didactic purposes, , the envelope of dimension value with the radius of 5m is also shown in Table IV, from the axis of the highest and lowest LPS, however this distance can be any one, since it is less than the distance from the critical point.

The mathematical approach methodology also allows for calculating the quota for a higher LPS, starting with the smallest height dimension of LPS.

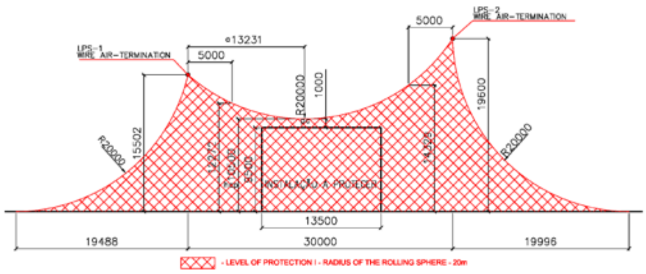

Fig. 8. Graphic representation of Col.1, of TABLE IV

B. LPS for one installation with three different heights

This example shows the result of the calculations of the one LPS, for one installation, with three different dimensions of height, width and layout configuration. This combination aims to show the potential of the mathematical approach methodology in the design of the LPS. The input data are: dimension of the installations, the installation distances in relation to the axes of LPS; coverage margin; Protection level (for Level I - radius of 20.0m) and distance between the LPS. The calculation results are shown in Table V and the graphic representations for columns: Col. 1 Col. 2 Col.3 and Col. 4 are respectively: Fig. 9, Fig.10, Fig. 11 and Fig.12.

TABLE V. RESULTS OF THE DIMENSION CALCULATIONS FIG. 8

Designation	**Layout 1**		**Layout 2**	
	Col. 1	Col.2	Col. 3	Col. 4
Height of the Installation 1 (h1)	12,00	12,00	7,50	7,50
Radius of h1 to the axis of the LPS-2 (r1)	4,83	4,83	8,00	8,00
Distance of h1 to the axis of the LPS-2	1,00	1,00	1,00	1,00
Height of the Installation 2 (h2)	8,50	8,50	6,00	6,00
Radius of h2 to the axis of the LPS-2 (r2)	12,50	12,50	14,17	14,17
Width of the Installation 2	5,88	5,88	5,60	5,60
Height of the Installation 3 (h3)	7,00	7,00	8,50	8,50
Radius of h3 to the axis of the LPS-1 (r1)	15,10	15,10	13,00	13,00
Distance of h3 to the axis of the LPS-1	1,00	1,00	1,00	1,00
Coverage margin design	1,00	1,00	1,00	1,00
Distance from the critical point of Lowest LPS (a)	12,794	15,000	10,235	15,000
Height of the critical point (hpc)	8,966	10,373	8,381	9,405
Height of LPS-1 calculated	13,594	17,144	25,321	16,176
Height of LPS-2 calculated	18,770	17,144	11,198	16,176
Coverage margin Installation 1 (cm1)	1,000	1,000	1,000	2,996
Height of the nearest point envelopment's edge h1 (hc1)	12,808	12,875	8,494	10,354
Radius from the nearest point of envelopment's edge h1 (rc1)	5,419	5,314	8,106	8,912
Coverage margin Installation 2 (cm2)	1,000	2,016	2,381	3,419
Height of the nearest point of envelopment's edge h2 (hc2)	9,475	10,502	8,381	9,417
Radius from the nearest point of envelopment's edge h2 (rc2)	12,724	12,729	10,235	14,295
Coverage margin Installation 3 (cm3)	1,966	3,373	1,000	1,000
Height of the nearest point of envelopment's edge h3 (hc3)	8,966	10,373	9,447	9,495
Radius from the nearest point of envelopment's edge h1 (rc3)	12,794	15,000	13,322	13,095

As noted, the methodology for this layout configuration and dimensions of installation presents two solutions with parallel wire air-termination: one with different heights (Col. 1 and Col.3) and another for the same height (Col.2 and Col. 4). The solution for a different quota level is the one whose design coverage margin is met for at least 2 installations and the third greater coverage margin. The solution for leveled LPS search only guarantees coverage margin for the most critical installation, while the other will stay with a coverage margin of an established design.

Depending on the layout configuration, a solution with LPS is only possible in the same quota. This occurs when the distance between the LPS, minus the distance from the critical point from lowest LPS (a) is greater than the radius of the rolling sphere of the protection level considered in design. In this case, the critical point is located in the half of the space between the LPS and the penetration of the radius of the sphere is calculated according to equation (1).

Fig. 9 (solution in different height - Layout 1 - Col. 1 of Table V) and Fig. 10 solution at the same height - Layout 1 - Col. 2 of Table V).

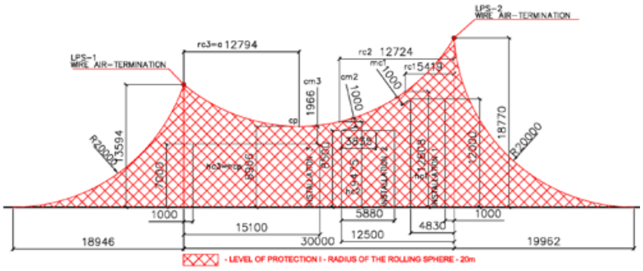

Fig. 9. Graphic representation of Col.1, of TABLE V

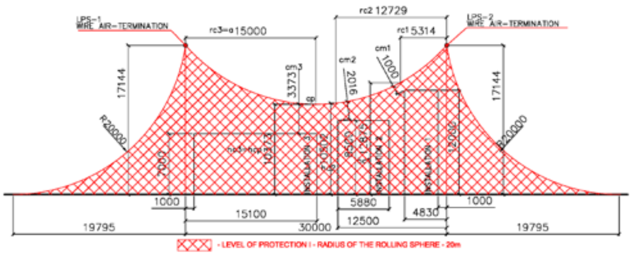

Fig. 10. Graphic representation of Col.2 of TABLE V

Fig. 11 (solution in different height - Layout 2 - Col. 3 of Table V) and Fig. 12 (solution in same height - Layout 2 - Col. 4 of Table V).

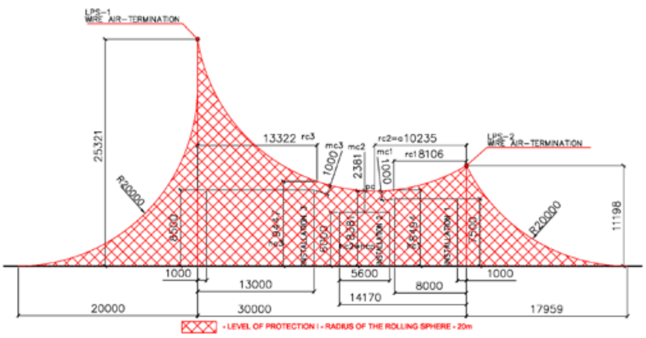

Fig. 11. Graphic representation of Col.3 of TABLE V

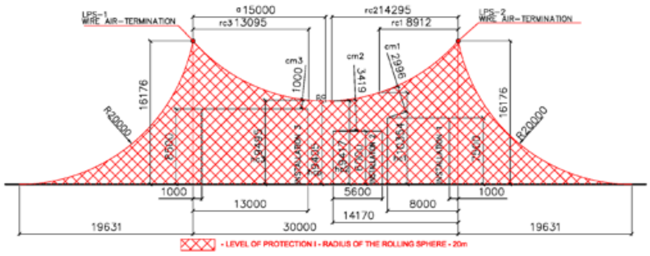

Fig. 12. Graphic representation of Col.4 of TABLE V

VII. Limits Dimensioning for LPS

A. Side limits of LPS protection levels

From deductions of the equations (2) and (3), we obtain the calculation result of the protection height limits of a LPS. In this example, the limits were calculated from 10.0m of the LPS axis and points where each level of protection touches the ground (surface coverage limits). For these calculations, we take the LPS of Fig.8 as an example, resulting in the data shown in Table VI and the graphic representation in FIG.13.

TABLE VI. Results of the LpS Side Limits Calculations

Designation	**LPS – Fig. 8**
Height LPS (m)	15.50
Height of limit at Level II, for Level I is zero (ground surface)	0.775
Height of limit at Level III, when the Level I is zero (ground surface)	2.399
Height of limit at Level IV, when the Level I is zero (ground surface)	3.706
Height of limit at Level III, when the Level II is zero (ground surface)	0.667
Height of limit at Level IV, when the Level II is zero (ground surface)	1.652
Height of limit at Level IV, when the Level III is zero (ground surface)	0.328
Height limit to Level I, when the distance is 10.0m of the LPS axis	2.394
Height limit to Level II, when the distance is 10.0m of the LPS axis	4.791
Height limit to Level III, when the distance is 10.0m of the LPS axis	6.924
Height limit to Level IV, when the distance is 10.0m of the LPS axis	8.183

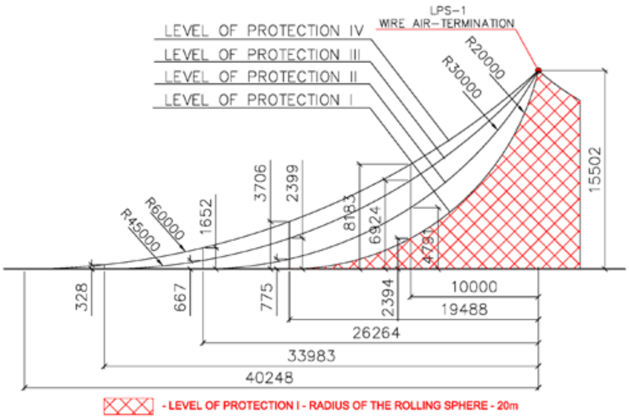

Fig. 13. Graphic representation of TABLE VI cross-section

B. Limits for the LPS of Fig.8

This approach aims to provide data that can support decision-making to review the design, if a need occurs in the implementation to: reduce the height of LPS; increase the gap between the LPS; there is a difference in the ground height between the LPS; there is need to increase the height of the structure/installation. The calculation result of the distance of the critical point to the lowest LPS (**a**) and the height of the critical point (**hcp**), according to the difference in the distance between ground level and distance between LPS is shown in Table VII.

TABLE VII. CALCULATIONS OF THE LPS LIMITS FOR FIG. 8

Designation		**Distance between LPS)**					
		30.0 (m)		**30.5 (m)**		**31.0 (m)**	
Reduce level	**Height of the lower LPS**	**a**	**hpc**	**a**	**hpc**	**a**	**hpc**
-	16.00	13.23	10.50	13.55	10.21	13.87	9.92
0.50	15.50	13.03	10.18	13.35	9.89	13.68	9.60
1.00	15.00	12.83	9.85	13.16	9.56	13.49	9.27
1.50	14.50	12.63	9.51				

The results of the calculated limits indicate that keeping the criterion of a 1.0m project for the coverage margin, the maximum distance between the LPS is 30.0m, since the height of the LPS is maintained, and defined in the design. However, considering the coverage margin between 0.1m to 1.0m (yellow cells) design provides adequate coverage, even if the spacing between the LPS is 31.0m and the difference is 0.5m between ground level. Red cells indicate that the envelope of the protection zone is lower for the installation or in the boundary condition of 0.1m.

In the event of needing an increase in these limits, the solution is to increase the height of the LPS of lower height, as it was considered that the cable captor of greater height was defined.

VIII. CONCLUSION

The use of the mathematical approach methodology for the analysis and design of LPS in a rolling sphere method provided important knowledge and security for issuing technical reports of the analyzed projects, as well as giving robustness to calculation reports to elaborate designs, making the graphical representations, and only a translation of the mathematical approach results.

This approach also allowed for subsidies for verifying the design's documents, because it allows us to identify possible deviations or development failures of this documentation, especially in the graphical part.

Finally, my experience left me convinced that improving the methodology through a more comprehensive and detailed mathematical modeling will provide an important tool for designers of such a system, since it will allow for the migration of mainly graphic designs for a mathematical basis.

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